ANOMALOUS SPECIFIC HEAT IN ULTRADEGENERATE QED AND QCD

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We discuss the origin of the anomalous $T \ln T^{-1}$ behavior of the low-temperature entropy and specific heat in ultradegenerate QED and QCD and report on a recent calculation which is complete to leading order in the coupling and which contains an infinite series of anomalous terms involving also fractional powers in T. This result involves dynamical hard-dense-loop resummation and interpolates between Debye screening effects at larger temperatures and non-Fermi-liquid behavior from only dynamically screened magnetic fields at low temperature.

1. Introduction

Cold dense quark matter has important deviations from Fermi-liquid behavior: below $T_c^{CSC} \sim 6$ to 60 MeV there is color superconductivity and long-range magnetic interactions are responsible for the fact that the energy gap is not proportional to $\exp(-c/g^2)$ as with short-range interactions, but instead to $\exp(-c'/g)$.[1,2] Above T_c^{CSC}, and for unpaired quarks also below, the only weakly (dynamically) screened magnetic interactions are also responsible for an anomalous behavior of entropy and specific heat, with a behavior $C_v \sim \alpha_s N_g N_f \mu^2 T \ln T^{-1}$ first discovered in the context of nonrelativistic QED by Holstein, Norton, and Pincus[3,4].

In QED this effect is probably unobservably small (though it may arise also from effective gauge field dynamics in systems of strongly correlated electrons), but in QCD it can be orders of magnitude larger because there are $N_g = 8$ gauge bosons instead of only one, and also α_s is much larger than α. However, more recently the existence of this effect had been questioned by Boyanovsky and de Vega[5], who instead found a $\alpha T^3 \ln T$ (when their renormalization-group summation of log's is undone).

In Ref.[6], we have recently confirmed the correctness of the original result and succeeded in calculating higher terms of the low-temperature expansion, for which only the coefficient of the leading log was known before.

The higher terms involve cubic roots of temperature, which can be traced to the fact that the frequency-dependent screening length of quasi-static magnetic modes is given by[7] $\kappa \simeq (\pi m_D^2 \omega/4)^{1/3}$, where m_D is the (electric) Debye mass.

2. Origin of the $T \ln T$ term

The non-Fermi-liquid behavior is usually discussed in terms of the spectral properties of the fermions. The leading fermionic contribution to the entropy density can be written as

$$\mathcal{S}_f \simeq -4NN_f \int \frac{d^4K}{(2\pi)^4} \frac{\partial n_f(\omega)}{\partial T} \left(\mathrm{Im}\, \ln S_+^{-1} + \mathrm{Im}\, \Sigma_+ \, \mathrm{Re}\, S_+ \right), \tag{1}$$

where it suffices to consider the particle (+) as opposed to antiparticle contribution. The $T \ln T$ behavior can then be obtained[3] from the singular behavior of the fermion self-energy at the Fermi surface[3,8,9],

$$\Sigma_+ \simeq \frac{g^2 C_f}{24\pi^2} (\omega - \mu) \ln \left(\frac{M^2}{(\omega-\mu)^2} \right) + i \frac{g^2 C_f}{12\pi} |\omega - \mu|. \tag{2}$$

However, it is not a priori justified to leave out the contributions from the gauge bosons. The entropy contributed by transverse modes can be written to two-loop accuracy as

$$\mathcal{S}_T \simeq -2N_g \int \frac{d^4K}{(2\pi)^4} \frac{\partial n_b(\omega)}{\partial T} \Big(\underbrace{\mathrm{Im}\, \ln D_T^{-1}}_{(A)} \underbrace{- \mathrm{Im}\, \Pi_T \, \mathrm{Re}\, D_T}_{(B)} \Big). \tag{3}$$

The fact that quasi-static transverse gauge bosons are only weakly screened by $\Pi_T \simeq -i\pi m_D^2 \omega/(4k)$ leads to nonanalytic behavior in T, which is exactly the same as that of the interaction part of S_f:

$$S_f^{\rm int} \simeq S_{(A)} \simeq -\frac{N_g m_D^2 T}{36} \ln T^{-1} \tag{4}$$

It is only because $\mathcal{S}_{(B)} \simeq -\mathcal{S}_{(A)}$ that S_f already gives the complete result to leading order.

If one organizes the calculation differently, as done in Ref.[5] where the specific heat is extracted from the internal energy, one in fact finds that all $T \ln T$ terms as contributed by the fermions cancel out. However, as we have shown in Ref.[10], the complete result is then coming from the internal energy of the gauge boson sector, explicitly neglected in Ref.[5], which resolves the contradiction with the original results.

3. Complete leading-order results

It turns out that it is in fact advantageous to reorganize the calculation such that all anomalous contributions come from the gauge boson sector, by integrating out the fermionic degrees of freedom first. This allows one to systematically calculate beyond the leading-log approximation without having to calculate the fermionic spectral densities beyond leading order. In Ref.[10] we have most recently shown that the infinite series of anomalous contributions is contained in the following hard-dense-loop (HDL)[11] resummed expression valid for $T \ll \mu$:

$$\frac{1}{N_g}(\mathcal{S}-\mathcal{S}^0) = -\frac{g_{\rm eff}^2\mu^2 T}{24\pi^2} - \frac{1}{2\pi^3}\int_0^\infty dq_0\,\frac{\partial n_b(q_0)}{\partial T}\int_0^\infty dq\,q^2\Bigg[2\,{\rm Im}\,\ln\left(\frac{q^2-q_0^2+\Pi_T^{\rm HDL}}{q^2-q_0^2}\right)+{\rm Im}\,\ln\left(\frac{q^2-q_0^2+\Pi_L^{\rm HDL}}{q^2-q_0^2}\right)\Bigg]+O(g_{\rm eff}^4\mu^2 T). \quad (5)$$

Here S^0 is the ideal-gas value of the entropy density, and $g_{\rm eff}^2 = g^2 N_f/2$ in QCD, but $e^2 N_f$ in QED.

At low temperature $T \ll g_{\rm eff}\mu$, one finds that the Landau damping cuts of the HDL propagators give rise to a series of the form

$$\frac{\mathcal{S}-\mathcal{S}_0}{N_g} = \frac{g_{\rm eff}^2\mu^2 T}{36\pi^2}\left(\ln\frac{4g_{\rm eff}\mu}{\pi^2 T} - 2 + \gamma_E - \frac{6}{\pi^2}\zeta'(2)\right) + c_1 T^{5/3} + c_2 T^{7/3} + c_3 T^3(\ln(g_{\rm eff}\mu/T)+c_4) + O(T^{11/3}), \quad (6)$$

where the coefficients up to and including c_4 can be found in Ref.[10].

At $g_{\rm eff}\mu \lesssim T \ll \mu$, one has also important quasiparticle contributions, most of which are invisible in the low-T expansion since they involve terms suppressed by factors of $e^{-m_D/\sqrt{3}T}$. For $g_{\rm eff}\mu \ll T \ll \mu$, one finally makes contact with more familiar results from thermal perturbation theory, as $\mathcal{S}-\mathcal{S}_0 \to N_g(-g_{\rm eff}^2\mu^2 T^2/(12\pi^2) + g_{\rm eff}^3\mu^3/(12\pi^4)+\ldots)$, where it is the longitudinal plasmon term $\propto m_D^3$ which makes (HDL) resummation necessary. The expression (5) thus interpolates between two physically rather different collective phenomena: the plasmon effect from (electric) Debye screening, which only arises at $T \gg g_{\rm eff}\mu$, and non-Fermi-liquid effects from the only dynamically screened magnetic interactions at $T \ll g_{\rm eff}\mu$, see Fig. 1.

4. Results for large and finite N_f

The above result (5) is the leading-order result for both QCD and QED at $T \ll \mu$. Higher-order terms either involve extra powers of g^2 or T^2/μ^2 (but not at the same time higher powers of $\ln T$)[4,12].

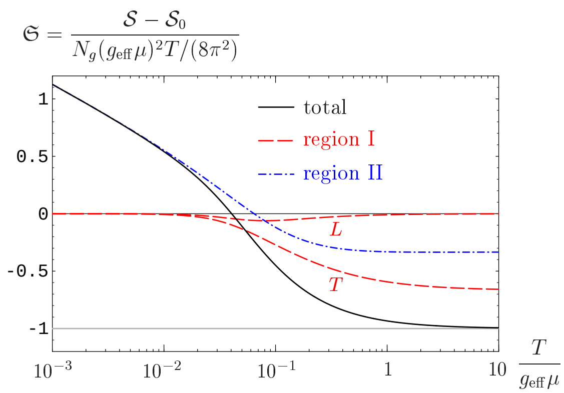

Figure 1. The function $\mathfrak{S}(T/(g_{\text{eff}}\mu))$ which determines the leading-order interaction contribution to the low-temperature entropy. The normalization is such that $\mathfrak{S} = -1$ corresponds to the result of ordinary perturbation theory. The dash-dotted line shows the contribution from spacelike momenta (region II), comprising HDL Landau damping and hard contributions; the two dashed lines give the transverse (T) and longitudinal (L) quasiparticle pole contributions (region I).

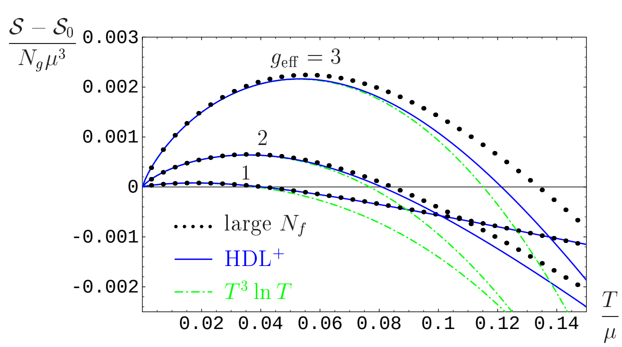

Figure 2. Complete entropy density in the large-N_f limit for the three values $g_{\text{eff}}(\bar{\mu}_{\text{MS}} = 2\mu) = 1, 2, 3$ (heavy dots), compared with the full HDL result (solid line). Also given is the low-temperature series up to and including the $T^3 \ln T$ contributions.

One case where we can actually investigate quantitatively the importance of higher-order terms is in the exactly solvable limit of large flavor number, which has been worked out for finite chemical potential in Ref. [13]. Figure 2 compares with the exact large-N_f result for $g_{\text{eff}}(\bar{\mu}_{\text{MS}} = 2\mu) = 1, 2, 3$, and we find that the HDL resummed result works well in the range where the entropy exceeds its ideal-gas value. The exact large-N_f result turns out to have even a slightly larger anomalous entropy.

In Fig. 3 we finally give results for the specific heat at finite N_f, which shows its anomalous behavior for a potentially interesting range in T/μ.

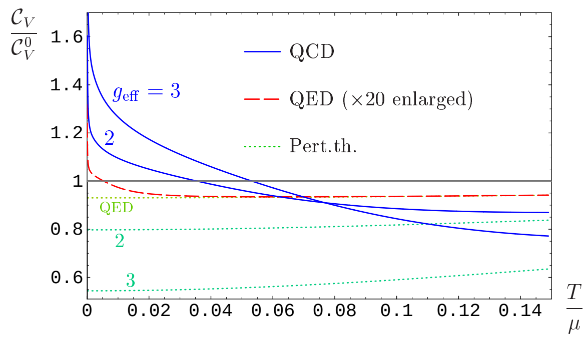

Figure 3. The HDL-resummed result for the specific heat $\mathcal{C}_v$, normalized to the ideal-gas value for $g_{\rm eff} = 2$ and 3 corresponding to $\alpha_s \approx 0.32$ and 0.72 in two-flavor QCD, and $g_{\rm eff} \approx 0.303$ for QED. The deviation of the QED result from the ideal-gas value is enlarged by a factor of 20 to make it more visible.

Acknowledgments

This work has been supported by the Austrian Science Foundation FWF, project no. 16387-N08 and by the ÖAD, project no. Amadée-16/2003.

References

1. D. T. Son, *Phys. Rev.* **D59**, 094019 (1999).
2. D. H. Rischke, *Prog. Part. Nucl. Phys.* **52**, 197 (2004).
3. T. Holstein, R. E. Norton and P. Pincus, *Phys. Rev.* **B8**, 2649 (1973).
4. S. Chakravarty, R. E. Norton and O. F. Syljuåsen, *Phys. Rev. Lett.* **74**, 1423 (1995).
5. D. Boyanovsky and H. J. de Vega, *Phys. Rev.* **D63**, 114028 (2001).
6. A. Ipp, A. Gerhold and A. Rebhan, *Phys. Rev.* **D69**, 011901 (2004).
7. H. A. Weldon, *Phys. Rev.* **D26**, 1394 (1982).
8. W. E. Brown, J. T. Liu and H.-c. Ren, *Phys. Rev.* **D61**, 114012 (2000).
9. C. Manuel, *Phys. Rev.* **D62**, 076009 (2000).
10. A. Gerhold, A. Ipp and A. Rebhan, hep-ph/0406087.
11. E. Braaten and R. D. Pisarski, *Nucl. Phys.* **B337**, 569 (1990); T. Altherr and U. Kraemmer, *Astropart. Phys.* **1**, 133 (1992); H. Vija and M. H. Thoma, *Phys. Lett.* **B342**, 212 (1995); C. Manuel, *Phys. Rev.* **D53**, 5866 (1996).
12. T. Schäfer and K. Schwenzer, hep-ph/0405053.
13. A. Ipp and A. Rebhan, *JHEP* **0306**, 032 (2003).